

Pumping two dilute gas Bose-Einstein condensates with Raman light scattering

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We propose an optical method for increasing the number of atoms in a pair of dilute gas Bose-Einstein condensates. The method uses laser-driven Raman transitions which scatter atoms between the condensate and non-condensate atom fractions. For a range of condensate phase differences there is destructive quantum interference of the amplitudes for scattering atoms out of the condensates. Because the total atom scattering rate into the condensates is unaffected the condensates grow. This mechanism is analogous to that responsible for optical lasing without inversion. Growth using macroscopic quantum interference may find application as a pump for an atom laser.

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In the recent experiments demonstrating Bose-Einstein condensation of alkali vapors the first stages of cooling are optical [1]. The final stage utilises evaporation of the hottest atoms out of the trap [2]. Despite the great success of evaporation it has the disadvantage of removing atoms from the system. Consequently alternative final stage cooling methods are being investigated. Velocity selective coherent population trapping (VSCPT) is one optical method potentially capable of cooling to the Bose-Einstein transition point [3].

In this paper we analyze an optical method for increasing the number of atoms in a pair of overlapping Bose-Einstein Condensates (BECs), such as have been produced in the laboratory using sympathetic cooling [4,5]. According to conventional spontaneous symmetry breaking arguments BECs are in coherent states with a definite global phase [6]. A pair of BECs therefore has a definite phase difference, which can be measured by a variety of techniques [7–11]. We utilise this phase difference to suppress transitions of atoms out of the condensates while driving transitions into the condensates, producing a net condensate growth. The transitions are driven by spontaneous Raman scattering of laser light [9]. For a certain range of condensate phase differences transitions out of the condensates are suppressed by destructive quantum interference. The interference is between the amplitudes for transitions from each condensate into the non-condensate atom fraction.

This suppression of atom scattering out of the condensates is analogous to the suppression of absorption that occurs in optical lasing without inversion [12]. In both cases the suppression is due to destructive quan-

tum interference between two channels to the same final state. This mechanism for condensate growth might also be suitable as an optically driven pump for an atom laser [13].

Spontaneous Raman scattering of laser light from two BECs has been analyzed by Ruostekoski and Walls [9], whose analysis we shall follow. They showed that the scattered light spectrum depends on the phase difference between the two condensates. The spectrum has two peaks, corresponding to transitions of atoms into and out of the condensates. For certain values of the condensate phase difference the second peak disappears due to the destructive quantum interference previously described. Although this suggests the possibility of condensate growth, only the short time behaviour was considered. In fact, the Raman lasers induce Josephson oscillations between the condensates [14]. We show that growth occurs and can persist over a complete period of this dynamics. However the necessary destructive interference only occurs for a particular range of condensate phase differences. Consequently, condensate growth only occurs for a subensemble of condensate pairs. Suitable subensemble members might be chosen after a measurement of the phase difference [7–11].

We consider the set-up first introduced by Javanainen [7]; a quantum degenerate gas with two ground states and a common excited state. All the ground state atoms are confined in the same trap. The pair of condensates are in two different Zeeman sublevels $|b\rangle = |g, m\rangle$ and $|c\rangle = |g, m-2\rangle$. The state $|c\rangle$ is optically coupled to the electronically excited state $|e\rangle = |e, m-1\rangle$ by the field \mathbf{E}_2 having a polarization σ_+ and frequency Ω_2 . Similarly, the state $|b\rangle$ is coupled to $|e\rangle$ by the field \mathbf{E}_1 with a polarization σ_- and frequency Ω_1 . Following Ref. [15] the Hamiltonian density for the system is

$$\begin{aligned} \mathcal{H} = & \psi_b^\dagger H \psi_b + \psi_c^\dagger (H + \hbar\omega_{cb}) \psi_c \\ & + \psi_e^\dagger (H + \hbar\omega_{eb}) \psi_e + \mathcal{H}_F \\ & - \left(\mathbf{d}_b \cdot \mathbf{E}_1 \psi_b^\dagger \psi_e + \mathbf{d}_c \cdot \mathbf{E}_2 \psi_c^\dagger \psi_e + \text{h.c.} \right). \end{aligned} \quad (1)$$

The first three terms reflect the center of mass energy, H , and the internal energies of the atoms in the absence of electromagnetic fields. The frequencies for the optical transitions $e \leftrightarrow b$ and $e \leftrightarrow c$ are ω_{eb} and ω_{ec} ($\omega_{cb} = \omega_{eb} - \omega_{ec}$), respectively. \mathcal{H}_F is the Hamiltonian density for the free electromagnetic field. The final, bracketed, terms are for the atom-light dipole interaction. The dipole matrix

element for the atomic transition $e \leftrightarrow b$ ($e \leftrightarrow c$) is given by \mathbf{d}_b (\mathbf{d}_c).

We assume that the driving light fields \mathbf{E}_{di}^+ are in coherent states and detuned far from single photon resonance so that multiple scattering can be ignored. We also assume they are plane waves propagating in the positive z direction with wavevectors $\boldsymbol{\kappa}_i$

$$\tilde{\mathbf{E}}_{d1}^+(\mathbf{r}) = \frac{1}{2}\mathcal{E}_1\hat{\mathbf{e}}_- \exp(i\boldsymbol{\kappa}_1 \cdot \mathbf{r}), \quad (2a)$$

$$\tilde{\mathbf{E}}_{d2}^+(\mathbf{r}) = \frac{1}{2}\mathcal{E}_2\hat{\mathbf{e}}_+ \exp(i\boldsymbol{\kappa}_2 \cdot \mathbf{r}), \quad (2b)$$

where $\hat{\mathbf{e}}_i$ are the unit circular polarization vectors, and we have defined slowly varying fields by $\tilde{\mathbf{E}}_i^+ = e^{i\Omega_i t}\mathbf{E}_i^+$. We also define the slowly varying matter field $\tilde{\psi}_c = e^{i(\Omega_1 - \Omega_2)t}\psi_c$.

In the limit of large detuning the excited state field operator ψ_e may be eliminated adiabatically. Following Javanainen and Ruostekoski [15] the scattered electric fields may then be expressed in terms of the driving fields as $\tilde{\mathbf{E}}_s^+ = \tilde{\mathbf{E}}_{s1}^+ + \tilde{\mathbf{E}}_{s2}^+$ where $\tilde{\mathbf{E}}_{s1}^+$ is radiated by decays into state $|b\rangle$,

$$\begin{aligned} \tilde{\mathbf{E}}_{s1}^+(\mathbf{r}, t) &= \int d^3r' \mathbf{K}(\mathbf{d}_b)\psi_b^\dagger\psi_e \\ &= \frac{1}{\hbar\Delta_1} \int d^3r' \mathbf{K}(\mathbf{d}_b) \\ &\quad \times \left\{ \mathbf{d}_b \cdot \tilde{\mathbf{E}}_{d1}^+ \psi_b^\dagger\psi_b + \mathbf{d}_c \cdot \tilde{\mathbf{E}}_{d2}^+ \psi_b^\dagger\tilde{\psi}_c \right\}. \end{aligned} \quad (3)$$

The driving fields and atom fields are all functions of \mathbf{r}' and t . $\Delta_1 = \Omega_1 - \omega_{eb}$ is the atom-field detuning of field 1. The first line represents the radiation from the atomic dipole density, and the second follows after adiabatic elimination of the excited state. $\tilde{\mathbf{E}}_{s2}^+$, which is radiated by decays into state $|c\rangle$, is found by swapping subscripts b and c and swapping the driving fields $\tilde{\mathbf{E}}_{d1}^+$ and $\tilde{\mathbf{E}}_{d2}^+$. We have used the first Born approximation based on the assumption that the incoming fields dominate inside the sample, as multiple scattering is negligible. The kernel $\mathbf{K}(\mathbf{d})$ is the familiar expression [16] for the positive-frequency component of the electric field at \mathbf{r} from a monochromatic dipole with the complex amplitude \mathbf{d} , located at \mathbf{r}' .

After adiabatic elimination of the excited state from the Hamiltonian density Eq. (1), and approximation of the electric fields by the driving fields, the following Hamiltonian density is found to first order in the inverse atom-field detuning [9]

$$\begin{aligned} \mathcal{H}_M &= \psi_b^\dagger(H - \hbar\delta_1)\psi_b + \tilde{\psi}_c^\dagger(H - \hbar\delta_{cb} - \hbar\delta_2)\tilde{\psi}_c \\ &\quad + \hbar\kappa \left(\psi_b^\dagger\tilde{\psi}_c \exp(-i\boldsymbol{\kappa}_{12} \cdot \mathbf{r}) + \tilde{\psi}_c^\dagger\psi_b \exp(i\boldsymbol{\kappa}_{12} \cdot \mathbf{r}) \right), \end{aligned} \quad (4)$$

where $\boldsymbol{\kappa}_{12} = \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2$ is the wavevector difference of the driving light fields. We have introduced the light-induced level shifts δ_i , the detuning from two-photon resonance

$\delta_{cb} = \Omega_1 - \Omega_2 - \omega_{cb}$, and the Raman coupling coefficient κ

$$\delta_1 = \frac{|\mathcal{E}_1|^2 d_b^2}{4\hbar^2 \Delta_1}, \quad \delta_2 = \frac{|\mathcal{E}_2|^2 d_c^2}{4\hbar^2 \Delta_1}, \quad \kappa = \frac{\mathcal{E}_1^* \mathcal{E}_2 d_b d_c}{4\hbar^2 \Delta_1}. \quad (5)$$

The dipole matrix elements d_b and d_c contain the reduced dipole matrix elements and the corresponding nonvanishing Clebsch-Gordan coefficients. To simplify the algebra, we assume κ to be real.

The intensity of the scattered light at position \mathbf{r} is given by

$$I(\mathbf{r}) = 2c\epsilon_0 \langle \tilde{\mathbf{E}}_s^- \cdot \tilde{\mathbf{E}}_s^+ \rangle. \quad (6)$$

Substituting in the expressions for the scattered fields in terms of the atom fields, Eq. (3), generates a sum of terms for the intensity of the form

$$\begin{aligned} 2c\epsilon_0 \left(\frac{1}{\hbar\Delta_1} \right)^2 \int d^3r' d^3r'' [\mathbf{K}(\mathbf{d}_b)']^* \cdot \mathbf{K}(\mathbf{d}_b)'' \\ \times (\mathbf{d}_b^* \cdot \tilde{\mathbf{E}}_{d1}^-)(\mathbf{d}_b \cdot \tilde{\mathbf{E}}_{d1}^+) \langle \psi_b^\dagger \psi_b' \psi_b'' \psi_b'' \rangle. \end{aligned} \quad (7)$$

The $'$ and $''$ respectively denote functional dependence on \mathbf{r}' and \mathbf{r}'' . We now assume that the driving fields have the same wavevectors, so that $\boldsymbol{\kappa}_{12} = \mathbf{0}$. The dynamics of the ground state fields $\tilde{\psi}_c$ and ψ_b follows from the Hamiltonian Eq. (4). We assume a translationally invariant and non-interacting Bose gas. The matter field operators are given by the familiar plane wave representations $\psi_b(\mathbf{r}t) = V^{-1/2} \sum_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{r}) b_{\mathbf{k}}(t)$ and $\tilde{\psi}_c(\mathbf{r}t) = V^{-1/2} \sum_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{r}) \tilde{c}_{\mathbf{k}}(t)$, where V is the mode volume. In the absence of light, the center of mass motion in both ground states satisfies the dispersion relation $\epsilon_{\mathbf{k}} = \hbar|\mathbf{k}|^2/2m$, with m the atomic mass. Defining the effective two-photon detuning $2\bar{\delta} = \delta_{cb} - \delta_1 + \delta_2$ and the condensate oscillation frequency $\Omega_R = (\bar{\delta}^2 + \kappa^2)^{1/2}$, the mode operators at time t are given in terms of the operators at time $t = 0$ by

$$\tilde{c}_{\mathbf{k}}(t) = e^{i\alpha t} \{ A\tilde{c}_{\mathbf{k}}(0) - Bb_{\mathbf{k}}(0) \}, \quad (8a)$$

$$b_{\mathbf{k}}(t) = e^{i\alpha t} \{ A^*b_{\mathbf{k}}(0) - B\tilde{c}_{\mathbf{k}}(0) \}, \quad (8b)$$

$$\alpha = \bar{\delta} + \delta_1 - \epsilon_{\mathbf{k}}, \quad (8c)$$

$$A = \cos \Omega_R t + \frac{i\bar{\delta}}{\Omega_R} \sin \Omega_R t, \quad B = i \frac{\kappa}{\Omega_R} \sin \Omega_R t. \quad (8d)$$

Before the light is switched on, the atoms in the states $|b\rangle$ and $|c\rangle$ are assumed to be uncorrelated. The Raman fields induce a coupling between the two levels. In the presence of Bose condensates in the ground states, the coupling between the two condensates is analogous to the coherent tunneling of Cooper pairs in a Josephson junction [7,14]. The expectation values of products of four atom field operators, such as occurs in Eq. (7), may be evaluated after substituting in the expressions Eq. (8) for the fields at time t in terms of the time zero fields. For example the expectation value in Eq. (7) becomes

$$\begin{aligned} & \langle \psi_b^{\dagger'} \psi_b' \psi_b^{\dagger''} \psi_b'' \rangle|_t = \\ & \langle [A\psi_b^{\dagger'} + B\psi_c^{\dagger'}][A^*\psi_b' - B\psi_c'] \\ & \times [A\psi_b^{\dagger''} + B\psi_c^{\dagger''}][A^*\psi_b'' - B\psi_c''] \rangle, \end{aligned} \quad (9)$$

where all the field operators on the right hand side are evaluated at time zero.

The field operators are sums over the condensate and non-condensate modes. Since we are *only* interested in the change in the number of condensate atoms due to light scattering we need *only* evaluate those terms corresponding to scattering of atoms into or out of the condensate, *i.e.* the incoherent part of the scattering. We ignore scattering of atoms between non-condensate modes. Together with momentum conservation this leads to a considerable simplification of the terms like Eq. (9). Once a particular plane wave mode is chosen for the first factor in Eq. (9) the requirement for a non-zero expectation value determines the modes occurring in all the remaining factors. For example, the part of Eq. (9) relevant to condensate depletion and growth is

$$\langle D_0^\dagger D_- D_+^\dagger D_0 + D_+^\dagger D_0 D_+^\dagger D_+ \rangle, \quad (10)$$

where $D_i = A^*(t)b_i(0) - B(t)c_i(0)$ and the subscripts 0 and \pm respectively refer to the condensate mode and the non-condensate modes having momenta $\pm\hbar\Delta\kappa$. Here $\Delta\kappa = \Omega\hat{\mathbf{n}}/c - \kappa$ is the wavevector change of the scattered photon, and $\hat{\mathbf{n}} = \mathbf{r}/|\mathbf{r}|$ is the unit vector in the light scattering direction under consideration. The two non-condensate modes $+/-$ respectively arise from scattering of atoms into/out of the condensate. Note that the particular atomic mode denoted depends on the light scattering direction $\hat{\mathbf{n}}$, as does the polarization of the scattered light. In general, the polarizations of the emitted photons from the two different atomic transitions are not orthogonal. However, although the resulting interference terms are nonvanishing in a particular direction [9], their contribution to the total intensity of the scattered light vanishes after integration over all scattering directions. Because we are ultimately interested in the total intensity, we ignore the terms proportional to $\langle \hat{\mathbf{E}}_{s1}^- \cdot \hat{\mathbf{E}}_{s2}^+ \rangle + \text{c.c.}$ in the intensity of the scattered light.

For brevity we assume that the number of atoms in the ground non-condensate states are the same, $n_\pm = \langle b_\pm^\dagger b_\pm \rangle = \langle c_\pm^\dagger c_\pm \rangle$, and that $n_+ = n_- = n$ due to isotropy. Further simplification occurs if we assume that there are equal numbers of atoms N in each condensate, and that the laser intensities are chosen so that the level shifts are equal $\delta = \delta_1 = \delta_2$. Evaluating all the relevant terms in Eq. (6) we find the following expressions for the intensity due to scattering of atoms into and out of the condensates

$$I_{\text{in}} = 2Cn\delta d_s^2 \{N + 2\text{Re}[A^*B\langle c_0^\dagger b_0 \rangle]d_d^2\}, \quad (11)$$

$$I_{\text{out}} = 2C(n+1)\delta d_s^2 \{N + \text{Re}[(A^{*2} - B^2)\langle c_0^\dagger b_0 \rangle]\}, \quad (12)$$

$$C = \frac{c_L |\kappa|^4}{8\pi^2 \epsilon_0 \Delta_1} \frac{1}{|\mathbf{r}|^2} (1 - \frac{1}{2} \sin^2 \theta), \quad (13)$$

where c_L is the speed of light, $d_s^2 = d_b^2 + d_c^2$, and $d_d^2 = (d_b^2 - d_c^2)/d_s^2$. We next integrate these intensities over the Josephson oscillation period $P = 2\pi/\Omega_R$. We find the time averaged intensities

$$\frac{1}{P} \int_0^P I_{\text{in}} dt = C'n \left\{ 1 + \frac{\bar{\delta}\kappa}{\Omega_R^2} d_d^2 \cos \Theta \right\}, \quad (14)$$

$$\frac{1}{P} \int_0^P I_{\text{out}} dt = C'(n+1) \left\{ 1 + \frac{\kappa^2}{\Omega_R^2} \cos \Theta \right\}, \quad (15)$$

where $C' = 2C\delta d_s^2 N$, and Θ is the condensate phase difference. The non-condensate populations n are functions of $\Delta\kappa$ and hence of the scattering angle θ between $\hat{\mathbf{n}}$ and the laser propagation direction. Our final step is integration over all scattering directions. This yields the total scattered light intensity and hence the total atom transition rates. The angular integration has the effect of replacing n by

$$\frac{8\pi}{3} \tilde{n} \equiv 2\pi \int_0^\pi (1 - \frac{1}{2} \sin^2 \theta) n(\theta) \sin \theta d\theta, \quad (16)$$

and $n+1$ by $8\pi(\tilde{n}+1)/3$. This integral may be interpreted as the number of non-condensate atoms available for scattering into the condensates. In an infinite homogeneous system the integral is divergent at the low energy end. However, for a finite, trapped system a low energy cutoff is provided by the first excited state. A numerical integration assuming the Bose-Einstein distribution at $T = 400$ nK, and a low energy cutoff of $\hbar(2\pi \times 100)$ J, gives $(8\pi/3)\tilde{n} \approx 65$ for rubidium. However, this is a crude estimate since realistic systems are not expected to be in thermal equilibrium.

Assuming, for simplicity, equal dipole moments $d_d = 0$ the net rate of scattering of atoms into the condensates (atoms per second) is then, from Eqs. (14-16),

$$\begin{aligned} R = & -6\pi \left(\frac{\gamma}{\Delta_1} \right)^2 \left(\frac{I_d}{\hbar c_L |\kappa|^3} \right) N \\ & \times \left\{ 1 + \frac{\kappa^2}{\Omega_R^2} \cos \Theta [1 + \tilde{n}] \right\}, \end{aligned} \quad (17)$$

where $\gamma = d_b^2 |\kappa|^3 / (3\pi\epsilon_0 \hbar)$ is the free space spontaneous emission rate, and I_d is the intensity of the lasers. Condensate growth corresponds to a positive rate R . Assuming that $\tilde{n} \gg 1$ this is equivalent to the following requirement on the condensate phase difference

$$\cos \Theta < -\frac{1}{\tilde{n}} \frac{\Omega_R^2}{\kappa^2}. \quad (18)$$

This inequality is fulfilled by particular negative values of $\cos \Theta$ provided that \tilde{n} is sufficiently large, and that the effective two-photon detuning $2\bar{\delta}$ is sufficiently small. The latter may be chosen small by manipulating the light-induced level shifts or the relative frequency of the driving light beams. The assumption of equal wave numbers for the driving light fields in the calculations is not very

restrictive for the relative frequency, because an atom trap introduces an uncertainty for the momentum conservation. In the limit of small two-photon detuning the effective linewidth of the transition $c \rightarrow b$ may have an effect. However, it may be shown to be proportional to Δ_1^{-2} or smaller.

With a $1\text{ }\mu\text{m}$ wavelength, 1 mW cm^{-2} laser intensity, and $(\kappa^2/\Omega_R^2)\cos\Theta = -1/2$, the growth rate Eq. (17) is

$$R \approx N\tilde{n} \left(\frac{\gamma}{\Delta_1} \right)^2 (10^7\text{s}^{-1}). \quad (19)$$

A laser detuning of $\Delta_1/\gamma = 10^3$, condensates with $N = 10^3$, and $(8\pi/3)\tilde{n} = 65$ give a condensate growth rate of $R \approx 7 \times 10^4$ atoms per second. This rate is large enough to be useful for both atom laser pumping and for condensate growth. However, sustained growth will require repopulation of the relevant non-condensate atom modes by atom scattering processes. Other limitations on growth include diffusion of the condensate phase difference due to atom-atom interactions [17,18], and heating of the non-condensate atom fraction by Raman transitions. It is also required that the spatial overlap of the condensate wave functions is significant.

Multiple scattering can be ignored provided that only a small fraction of the incident photons are scattered. If the cross sectional area of the condensate is \mathcal{A} , then this is true, if $\mathcal{A} \gg 6\pi(\gamma/\Delta_1)^2 N\tilde{n}/|\kappa|^2$. With the preceding parameters this becomes $\mathcal{A} \gg 4 \times 10^{-3}\mu\text{m}^2$, which is easily fulfilled.

In the calculations we used the conventional spontaneous symmetry breaking arguments that BECs are in coherent states. However, this convenient approach is by no means necessary. In fact, for the present atomic level scheme the relative phase between the two condensates has been established in stochastic simulations of the measurements of spontaneously scattered photons, even though the condensates are initially in pure number states [11]. Without any measurements of the condensate phase difference the macroscopic quantum coherence is expected to undergo collapses and revivals [18]. Because the detections of spontaneously scattered photons establish the relative phase, they could also stabilize the phase against the collapse of the macroscopic wave function.

We have shown that quantum mechanical interference enables growth of two Raman driven Bose condensates. The growth mechanism is analogous to that for gain in the laser without inversion. In that case destructive quantum mechanical interference suppresses absorption, allowing gain to dominate. In our case interference suppresses scattering of atoms out of the condensates, allowing scattering of atoms into the condensates to dominate.

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